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An Empirical Comparison of Some Product Estimators*

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Abstract

In this paper, we undertake an extensive comparative study of some biased, almost unbiased and unbiased product estimators on the ground of different performance measures through Monte Carlo simulation that has not yet been initiated in the survey sampling literature. The simulation experiment is conducted using data on 20 natural populations available in the literature, and the performance indicators taken into consideration are the absolute relative bias, percentage relative efficiency, coverage rate of confidence intervals, standard deviation of the student t –statistic, and approach to symmetry (normality). This empirical study will not only facilitate to assess the overall relative performance of different competing product or product-type estimators but will also be beneficial to provide some guidelines towards further research in this direction.

Keywords: Auxiliary variable, prediction approach, product estimator.

1. Introduction

Let y_i and x_i $(i = 1, 2, \dots, N)$ be the values of the survey variable y and an auxiliary variable x on the ith unit of a finite population U of N units. With the aim of estimating unknown population mean $\bar{Y}$ of y at the moment that the population mean $\bar{X}$ of x is known, assume that a random sample s of n units is taken from U in accord with simple random sampling without replacement (SRSWOR). Let $\bar{y} = \frac{1}{n}\sum_{i \in s} y_i$ and $\bar{x} = \frac{1}{n}\sum_{i \in s} x_i$ be the sample means, $s_y^2 = \frac{1}{n-1}\sum_{i \in s}(y_i - \bar{y})^2$ and $s_x^2 = \frac{1}{n-1}\sum_{i \in s}(x_i - \bar{x})^2$ be the sample variances, and $s_{yx} = \frac{1}{n-1}\sum_{i \in s}(y_i - \bar{y})(x_i - \bar{x})$ be the sample covariance. Ordinarily, many survey statisticians do not give product method of estimation as much emphasis

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as ratio method of estimation. Because, they are on the opinion that occurrence of negatively correlated auxiliary variables is a rare phenomenon. But, in the context of sample surveys, it is not very uncommon to observe negatively correlated variables [see for example, (Chaubey et al., 1990; R. Sahoo et al., 2022)].

The classical product estimator of the population mean $\overline{Y}$ is defined by

$$\ell_P = \frac{\bar{y}\bar{x}}{\overline{X}} \tag{1}$$

[*cf.*, (Murthy, 1964)] which performs better than the mean per unit estimator $\bar{y}$ when $\rho C_y/C_x < -1/2$, where C_y and C_x are the coefficients of variation of y and x respectively, and ρ is the coefficient of correlation between them. Customarily, ℓ_P is biased having exact bias expression

$$B(\ell_P) = E(\ell_P) - \overline{Y} = \theta \frac{S_{yx}}{\overline{X}}\ , \tag{2}$$

where $\theta = \frac{N-n}{Nn}$ and $S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)\left(x_i - \overline{X}\right)$. But estimating and correcting bias by the accustomed method, (Robson, 1957) composed an unbiased estimator defined by

$$\ell_{RP} = \ell_P\left(1 - \theta c_{yx}\right) \tag{3}$$

where $c_{yx} = s_{yx}/\bar{y}\bar{x}$. Comparing variance expressions up to terms of order n^{-2}, (V. Srivastava et al., 1981) established that ℓ_{RP} is better than ℓ_P. On the other hand, under finite and infinite population's set-up, (Chaubey et al., 1990) proved that ℓ_{RP} is more efficient than ℓ_P if $\rho^2 > (n-2)^{-1}$.

Following (L. Sahoo, 1983) work, (Singh, 1989) constructed an almost unbiased product estimator (unbiased up to terms of $O(n^{-1})$) of the form.

$$\ell_{SP} = \ell_P/\left(1 + \theta c_{yx}\right) \tag{4}$$

Note that the estimators ℓ_{RP} and ℓ_{SP} are virtually equivalent in the sense that they rely on the same statistics and moreover their variances are equal to $O(n^{-2})$.

Under the prediction viewpoint [*c.f.*, (Basu, 1971; Bolfarine & Zacks, 2012), p.12], recently (Basu, 1971) and (R. Sahoo et al., 2022) formulated an almost unbiased product estimator defined by

$$\ell_{AP} = \frac{1}{\overline{X}}\left[\bar{y}\bar{x} - \theta\frac{N-2}{N-1}s_{yx} - \frac{1}{N-1}b_{yx}\left(\bar{x} - \overline{X}\right)^2\right] \tag{5}$$

where $b_{yx} = s_{yx}/s_x^2$. The authors undertook a comparative study of ℓ_P, ℓ_{RP} and ℓ_{AP} in the light of mean square errors up to $O(n^{-2})$. However, this is not sufficient to establish relative merit of an estimator for its optimal use in survey operations as a variety of other performance criteria and a variety of other estimators are available in the literature. In this paper, we concentrate on an empirical comparison of several product estimators using various performance criteria. But, before proceeding further, first we turn to the construction of an alternative product estimator borrowing the new idea introduced in (R. Sahoo et al., 2022).

2. An Alternative Almost Unbiased Product Estimator

To estimate $B(\ell_P)$ under prediction methodology, we need a suitable predictor for the unknown population covariance S_{yx}. Hence, let us write

$$(N-1)S_{yx} = (n-1)s_{yx} + (N-n-1)S_{yx(r)} + (1-f)n(\bar{y}-\bar{Y}_r)(\bar{x}-\bar{X}_r) \qquad (6)$$

where $r = U - s$ denotes the collection of un-sampled units of U, $f = \frac{n}{N}$, $\bar{Y}_r = \frac{1}{N-n}\sum_{i\in r} y_i$, $\bar{X}_r = \frac{1}{N-n}\sum_{i\in r} x_i$ and $S_{yx(r)} = \frac{1}{N-n-1}\sum_{i\in r}(y_i - \bar{Y}_r)(x_i - \bar{X}_r)$.

In equation (6), note that s_{yx} and $\bar{X}_r = \frac{N\bar{X}-n\bar{x}}{N-n}$ are known quantities whereas $\bar{Y}_r$ and $S_{yx(r)}$ are unknown. Hence, prediction of $(N-1)S_{yx}$ needs simultaneous prediction of $\bar{Y}_r$ and $S_{yx(r)}$ from the sample data. Letting M_r and C_r as their respective predictors, a predictor of S_{yx} can be created from the following equation:

$$(N-1)\hat{S}_{yx} = (n-1)s_{yx} + (N-n-1)C_r + (1-f)n(\bar{y}-M_r)(\bar{x}-\bar{X}_r). \qquad (7)$$

Inspired by the arguments given in (Basu, 1971) and (Sampford, 1978), and encouraged by (R. Sahoo et al., 2022), here we also rely on the tools of the classical estimation theory to find out a suitable predictor for $\bar{Y}_r$. Hence, use of s_{yx} and $\bar{y}\bar{x}/\bar{X}_r$ as predictors of $S_{yx(r)}$ and $\bar{Y}_r$ respectively *i.e.*, $C_r = s_{yx}$ and $M_r = \bar{y}\bar{x}/\bar{X}_r$ yields the following predictive estimator for S_{yx}:

$$H_{yx} = \frac{N-2}{N-1}s_{yx} - \frac{nN}{(N-1)}\left(\frac{\bar{y}}{N\bar{X}-n\bar{x}}\right)(\bar{x}-\bar{X})^2 \qquad (8)$$

This is of course biased for S_{yx}. However, estimation of $B(\ell_P)$ by $\theta\frac{H_{yx}}{\bar{X}}$ gives a predictive product estimator for $\bar{Y}$ defined by

$$\ell_{NP} = \ell_P - \frac{\theta}{\bar{X}}\left[\frac{N-2}{N-1}s_{yx} - \frac{nN}{N-1}\left(\frac{\bar{y}}{N\bar{X}-n\bar{x}}\right)(\bar{x}-\bar{X})^2\right] \qquad (9)$$

3. Estimators Considered for the Simulation Study

We consider some more estimators along with those considered/proposed above for our empirical investigation (simulation study). This will make our comparative study more productive. But, the task of searching for all product estimators available in the survey sampling literature is not attainable. However, we present below a brief review of some product estimators as easily accessible comfortably in the literature.

(Gupta & Adhvaryu, 1982) constructed an unbiased product estimator after correcting bias of the mean of the product estimator $\bar{p}/\bar{X}$, where $\bar{p} = \frac{1}{n}\sum_{i\in s} y_i x_i$. The estimator is given by

$$\ell_{GA} = \frac{1}{\bar{X}}\left[\bar{p} - \frac{N-1}{N}s_{yx}\right] \qquad (10)$$

But, this estimator is not considered as it is equivalent to (Robson, 1957) unbiased estimator ℓ_{RP} and therefore both estimators yield same numerical results in respect of different performance measures taken into account.

(S. Srivastava, 1983) defined a predictive product estimator of the form

$$\ell_S = \bar{y}\frac{n\bar{X}+(N-2n)\bar{x}}{N\bar{X}-n\bar{x}} \tag{11}$$

After adjusting ℓ_S for the bias, (Quenouille, 1956) and (J. Sahoo & Sahoo, 1999) formulated an almost unbiased predictive product estimator defined by

$$\ell_{SS} = \ell_S - \theta\frac{\bar{y}\bar{x}}{N\bar{X}-n\bar{x}}\left[(N-n)c_{yx} + nc_x^2\right] \tag{12}$$

where $c_x^2 = s_x^2/\bar{x}^2$.

Following (Quenouille, 1956), (Shukla, 1976), developed an almost unbiased product estimator that is based on the random splitting of the sample into two equal parts. But, for our purpose we do not prefer this estimator as it needs sample sizes of even numbers and moreover, as shown by (V. Srivastava et al., 1981) it is uniformly less efficient than ℓ_P and ℓ_{RP}. On the similar grounds we also exclude an additional almost unbiased product estimator created in Sahoo & Sahoo (1999).

Finally, we have taken seven estimators *viz*., $\ell_P, \ell_{RP}, \ell_{SP}, \ell_{AP}, \ell_{NP}, \ell_S$ and ℓ_{SS} for the purpose of the present simulation study. An interesting common feature of all the seven estimators is that their variance/MSE expressions are equal to terms of order n^{-1} as is given by

$$V(\ell) = \theta\left(S_y^2 + 2RS_{yx} + R^2S_x^2\right) \tag{13}$$

where ℓ stands for any one of the said estimators, $R = \overline{Y}/\overline{X}$, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^2$ and $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \overline{X}\right)^2$.

4. Performance of the Selected Estimators

After selecting seven competing estimators $\ell_P, \ell_{RP}, \ell_{SP}, \ell_{AP}, \ell_{NP}, \ell_S$ and ℓ_{SS}, the next important task is to analyze their relative performance on various grounds. However, we would like to remark here that even if our sampling method is simple *i.e*., SRSWOR, the estimators are not only non-linear functions of some statistics but some of them are also very complex. Due to complex structure of the estimators, derivation of exact results on their design-based bias, variance/MSE and other performance measures under a finite population set up is not straightforward and finally, a theoretical comparison of their performances is not practicable. On the other hand, we also see that the asymptotic results are not simple enough to rely for drawing valid conclusions regarding relative merits of different estimators. Moving from these considerations, an effort has been made here to compare the performances of the proposed estimators empirically through calculations of values for different performance measures. A *Monte Carlo Simulation* exercise is carried out by drawing a series of 5000 independent samples of sizes 3, 5 and 7 from 20 populations given in some standard textbooks.

Qualities of the eight comparable estimators are assessed on the basis of five performance measures *viz*., absolute relative bias, percentage relative efficiency, coverage rate of confidence intervals, standard deviation of the student t −statistic, and approach to symmetry. For an estimator ℓ, these measures are explained as follows:

1. *Absolute Relative Bias (ARB)*: As the bias of an estimator is either negative or positive, for comparison purpose we consider its absolute value. This measure is given by

$$ARB(\ell) = \left|\frac{Bias\ (\ell)}{\bar{Y}}\right| = \left|\frac{E(\ell) - \bar{Y}}{\bar{Y}}\right| \tag{14}$$

2. *Percentage Relative Efficiency (PRE):* For simplicity, we compute relative efficiency of an estimator with respect to the simple expansion estimator $\bar{y}$ as defined by

$$PRE(\ell) = \frac{100 \times V(\bar{y})}{V(\ell)} \tag{15}$$

where $V(\bar{y}) = \theta S_y^2$ and $V(\ell) = E[\ell]^2 - [E(\ell)]^2$.

3. *Coverage Rate (CR) for* $\bar{Y}$ *Based on* $100(1-\alpha)\%$ (95% *or* 99%) *Confidence Interval:* When ℓ is a point estimator for $\bar{Y}$, a $100(1-\alpha)\%$ confidence interval for $\bar{Y}$ based on its variance estimator $\hat{V}(\ell)$ is given by

$$\ell \pm z_{1-\frac{\alpha}{2}}\sqrt{\hat{V}(\ell)} \tag{16}$$

where $z_{1-\frac{\alpha}{2}}$ is exceeded with probability $\alpha/2$ by the unit normal variate under the assumption that the sampling distribution of ℓ is approximately normal and

$$\hat{V}(\ell) = \theta\left[s_y^2 + 2rs_{yx} + r^2 s_x^2\right] \tag{17}$$

with $r = \bar{y}/\bar{x}$. Note that this is the common asymptotic expression for the estimated variance of the comparable estimators. This interval will contain the unknown mean $\bar{Y}$ for an approximate proportion of $100(1-\alpha)\%$ of repeated independent samples drawn from a given population if it is assumed that ℓ is asymptotically normally distributed with mean $\bar{Y}$ and variance $V(\ell)$.

4. *Standard Deviation of the Student* t *Statistic:* It is of interest to study the shape of the sampling distribution of the t −statistic by

$$t = \frac{|\ell - \bar{Y}|}{\sqrt{\hat{V}(\ell)}}. \tag{18}$$

If it is assumed that $\hat{V}(\ell)$ is a good estimate of $V(\ell)$, then for small n, the sampling distribution of t should conform closely to that of Student t variable with $n-1$ degree of freedom. The standard deviation (SD) of t defined by

$$\sigma_t = +\sqrt{[E(t - E(t))^2]} \tag{19}$$

symbolizes to what extent the estimator ℓ maintains its consistency for the sampling fluctuations. It is known that the SD for the Student t distribution with $n\ (> 2)$ degree of freedom is $+\sqrt{n/(n-2)}$.

5. *Approach to Symmetry (Normality):* To quantify asymmetry of the sampling distribution of ℓ, we consider β_1 coefficient (coefficient of skewness) given by

$$\beta_1 = \frac{\left[E(\ell - E(\ell))^3\right]^2}{[E(\ell - E(\ell))^2]^3} \tag{20}$$

whose value is zero for a symmetrical (normal) distribution. The more close the value of β_1 to zero, more is the symmetricity of the distribution of the estimator.

5. The Simulation Study

The simulation study reported here uses data of 20 natural populations. Source, size (N) and variables y and x for these populations are presented in table 1. 5,000 independent samples of sizes $n = 3,\ 5$ and 7, are drawn without replacement from each population. Based on the observed values of (y, x) of each realized sample, the estimates are calculated. Then, 5000 such values of an estimate may be termed as the empirical sampling distribution which closely approximates the exact sampling distribution that cannot be easily obtainable.

If $\ell^{(j)}$ denotes the value of the estimator ℓ for the jth sample, then we calculate

$$\frac{1}{5000}\sum_{j=1}^{5000} \ell^{(j)},\ \frac{1}{5000}\sum_{j=1}^{5000} \ell^{(j)^2} \text{ and } \frac{1}{5000}\sum_{j=1}^{5000}\left(\ell^{(j)} - V(\ell)\right)^2$$

as estimates of

$$E(\ell),\ E(\ell)^2 \text{ and } V(\ell) = E(\ell - V(\ell))^2$$

respectively. Then the measures ARB, PRE, σ_t and β_1 are computed in the usual manner. For each sample, the confidence interval is calculated and then counting the number of intervals that contain the true value of $\bar{Y}$, the coverage rates are finally calculated and expressed in percentage.

The simulated results in favor of the performance measures for all competing estimators and all populations are summarized in tables 2 to 16 and discussions on the empirical findings via simulation are briefly discussed in sub-sections 5.1 to 5.5. The entries for the best performer cases are boldly marked and those for the second best performer cases are underlined. But for CRs only the unique best performer cases are boldly marked.

Table 1: Populations under study

Pop. No.	Source	N	y	x
1	Gujarati & Porter (2009, p.406)	35 passenger cars (1-35)	average miles per gallon	engine HP
2	Gujarati & Porter (2009, p.406)	35 passenger cars (36-70)	average miles per gallon	engine HP
3	Gujarati & Porter (2009, p.51)	27 years	average hourly earnings	civilian labor force participation rate
4	Maddala (1992, p.194)	34 rural lands (1-34)	sale price of land	distance from airport
5	Maddala (1992, p.194)	33 rural lands (35-67)	sale price of land	distance from airport
6	Morrison (1990, p. 470)	26 lighter and heavier under wt. young males	pigment creatinine	phosphate (mg/mL)
7	(Bhuyan, 2008)	28 married couples of middle class families	fertility level (no. of ever born children)	education level of father (in completed years)
8	Bhuyan (2005, p.76)	28 two times milking cows	daily milk production	wt. of cow after lactation period
9	Bhuyan (2005, p.76)	28 three times milking cows	daily milk production	wt. of cow after lactation period
10	Johnson & Wichern, (2007, p.215)	20 healthy females	sweat rate	potassium content
11	Rawlings et al (1998, p.396)	20 plots (depth 1)	sand percentage	silt percentage
12	Rawlings et al (1998, p.396)	20 plots (depth 2)	clay percentage	sand percentage
13	Montgomery et al., (2012, p.556)	32 automobiles	miles/gallon	horsepower
14	Montgomery et al., (2012, p.15)	20 obs.	shear strength	age of propellant
15	Montgomery et al., (2012, p.291)	16 obs.	conversion of $n-$heptane to acetylene (%)	contact time (sec)
16	Montgomery et al., (2012, p.483)	20 time periods	selling price of toothpaste per pound	market share of toothpaste
17	Montgomery et al., (2012, p.572)	32 young red wines	quality rating (20 maximum)	total SO_2 (ppm)
18	Montgomery et al., (2012, p.558)	27 Belle Ayr liquefaction runs	oil yield	coal total
19	Rencher (2002, p.269)	23 obs. (1-23)	evaporation	minimum daily relative humidity
20	Rencher (2002, p.269)	23 obs.(24-46)	evaporation	minimum daily relative humidity

5.1 Results on the ARB

Numerical values on the ARB of the two biased estimators ℓ_P and ℓ_S, and four almost unbiased estimators ℓ_{SP}, ℓ_{AP}, ℓ_{NP} and ℓ_{SS} are shown in tables 2, 3 and 4. As is ordinarily expected, the ARB of an estimator more or less diminishes with the enlargement of sample size, and the ARB values of the biased estimators, except some few cases, greater than those values of the almost unbiased estimators. The predictive product estimator ℓ_S appears to be slightly less biased than the classical product estimator ℓ_P because of its better performance in 11, 13 and 14 cases for $n = 3, 5$ and 7 respectively. This also indicates that the bias of ℓ_S decreases more rapidly than that of ℓ_P with the increase in sample size.

Table 2: ARB of the estimators for $n = 3$

Pop. No.	ℓ_P	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.0024	0.0019	0.0003	0.0006	0.0014	0.0000
2	0.0160	0.0028	0.0007	0.0012	0.0108	0.0008
3	0.0047	0.0013	0.0004	0.0006	0.0043	0.0009
4	0.0472	0.0174	0.0098	0.0034	0.0318	0.0145
5	0.0435	0.0068	0.0007	0.0013	0.0352	0.0010
6	2.2228	0.9151	0.5643	0.4012	3.2412	0.3268
7	0.3412	0.2322	0.0202	0.0195	0.3636	0.0235
8	0.1610	0.1053	0.0005	0.0040	0.1548	0.0089
9	0.1582	0.1022	0.0231	0.0040	0.1521	0.0088
10	0.8741	0.2946	0.0714	0.5770	0.6781	0.3678
11	0.9123	0.0038	0.0069	0.2544	0.8633	0.6160
12	0.8622	0.0078	0.0065	0.0072	0.6269	0.5977
13	0.2250	0.1215	0.0034	0.0046	0.2164	0.0080
14	0.1004	0.0996	0.0001	0.0329	0.4276	0.0668
15	3.1634	1.9673	1.6669	1.4313	14.3109	1.5257
16	1.1903	0.6618	0.0383	0.5695	2.8898	0.1609
17	0.2125	0.0968	0.0036	0.0032	0.2007	0.0039
18	0.2444	0.0660	0.0045	0.0016	0.2842	0.0034
19	0.0119	0.0039	0.0004	0.0019	0.0040	0.0025
20	0.0149	0.0032	0.0015	0.0013	0.0047	0.0018

Table 3: ARB of the estimators for $n = 5$

Pop. No.	ℓ_P	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.0014	0.0004	0.0000	0.0003	0.0008	0.0001
2	0.0090	0.0004	0.0000	0.0007	0.0036	0.0005
3	0.0026	0.0007	0.0002	0.0000	0.0022	0.0001
4	0.0265	0.0030	0.0002	0.0023	0.0186	0.0107
5	0.0244	0.0013	0.0000	0.0007	0.0160	0.0003
6	1.2177	0.4369	0.1894	0.1435	1.7817	0.2887
7	0.1884	0.0292	0.0071	0.0098	0.2048	0.0067
8	0.0889	0.0186	0.0004	0.0026	0.0851	0.0045
9	0.0873	0.0181	0.0003	0.0026	0.0836	0.0044
10	0.4628	0.1194	0.0351	0.0438	0.3861	0.2636
11	0.8065	0.0005	0.0077	0.1229	0.7825	0.4325
12	0.0329	0.0047	0.0017	0.1043	0.6385	0.4272
13	0.1257	0.0253	0.0010	0.0030	0.1193	0.0043
14	0.0532	0.0077	0.0002	0.0201	0.0920	0.0904
15	4.6060	0.6133	0.0996	4.0103	5.4915	0.9876
16	0.6302	0.1985	0.0316	0.1964	1.6638	0.2042
17	0.1187	0.0209	0.0026	0.0020	0.1079	0.0014
18	0.1344	0.0228	0.0018	0.0110	0.1690	0.0020
19	0.0065	0.0004	0.0002	0.0010	0.0016	0.0016
20	0.0080	0.0002	0.0001	0.0014	0.0026	0.0030

Among the almost unbiased estimators, ℓ_{SP} turns out as the worst performer whereas ℓ_{AP} is superior to others as it is ranked either in the first or second or third place for all populations. It seems to be the least biased as its ARB is the minimum in 11, 15 and 14 populations for $n = 3, 5$ and 7 respectively. This shows that the performance of ℓ_{AP} improves gradually as the sample size gets larger. On the consideration of their positions in different populations and for different sample sizes, ℓ_{NP} and ℓ_{SS} can be regarded as the second least biased and third least biased estimators respectively.

Table 4: ARB of the estimators for $n = 7$

Pop. No.	ℓ_P	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.0008	0.0005	0.0000	0.0002	0.0004	0.0003
2	0.0051	0.0004	0.0000	0.0002	0.0005	0.0003
3	0.0014	0.0009	0.0006	0.0000	0.0011	0.0001
4	0.0149	0.0006	0.0002	0.0014	0.0043	0.0071
5	0.0136	0.0003	0.0000	0.0004	0.0053	0.0001
6	0.6523	0.1875	0.0080	0.0538	0.5603	0.1461
7	0.1024	0.0096	0.0057	0.0050	0.1155	0.0043
8	0.0483	0.0042	0.0004	0.0016	0.0458	0.0026
9	0.0475	0.0041	0.0003	0.0015	0.0450	0.0025
10	0.2314	0.0394	0.0255	0.2202	0.8669	0.1880
11	0.5133	0.0001	0.0083	0.0524	0.4648	0.1687
12	0.0165	0.0104	0.0032	0.0012	0.5803	0.0030
13	0.0698	0.0065	0.0002	0.0018	0.0647	0.0025
14	0.0266	0.0013	0.0003	0.0102	0.1197	0.0454
15	1.7300	0.2170	0.0775	0.1785	1.2776	0.1698
16	0.3151	0.0697	0.0249	0.0665	0.3080	0.2693
17	0.0659	0.0055	0.0015	0.0011	0.0558	0.0009
18	0.0726	0.0069	0.0008	0.0057	0.1044	0.0010
19	0.0064	0.0014	0.0000	0.0005	0.0015	0.0008
20	0.0042	0.0020	0.0001	0.0000	0.0025	0.0016

Table 5: PRE of the estimators for $n = 3$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	102.22	82.999	83.708	84.036	103.25	83.672	83.002
2	90.013	98.513	97.148	121.97	98.768	79.286	98.408
3	25.828	26.762	26.161	26.676	26.345	25.806	26.167
4	37.976	41.165	38.624	42.294	42.326	40.269	37.277
5	84.995	70.981	69.583	71.311	71.342	84.586	71.840
6	86.726	613.43	229.17	713.00	721.71	52.195	269.03
7	423.11	631.34	599.45	842.89	648.71	408.76	652.13
8	639.63	933.69	568.27	640.78	937.35	646.35	652.50
9	891.11	656.72	582.44	957.94	693.79	773.54	670.29
10	7.6668	22.423	15.433	21.068	4.8039	4.4501	6.0135
11	9.6833	9.4415	9.5883	9.6904	9.6902	9.1431	9.2282
12	5.4632	5.2687	5.3919	5.3993	5.3967	5.1024	5.1290
13	1136.8	777.86	696.15	1231.8	1185.3	1169.1	789.74
14	25.297	15.983	14.522	26.068	14.334	12.852	9.9440
15	22.788	233.87	64.956	245.83	95.950	90.040	23.689
16	33.408	141.23	77.656	139.38	73.183	10.391	197.82
17	731.55	1077.6	666.14	730.00	836.57	605.50	736.35
18	4.0554	4.0856	4.0789	4.0866	4.0858	4.0504	4.0818
19	28.186	28.717	28.677	28.701	28.439	28.517	28.415
20	27.387	29.132	28.944	30.187	28.969	28.050	28.282

Table 6: PRE of the estimators for $n = 5$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	78.367	66.838	67.059	67.258	79.154	66.869	66.762
2	83.634	83.763	83.607	97.403	96.061	84.807	83.777
3	21.462	21.676	21.677	21.744	21.665	21.444	21.680
4	33.802	37.809	32.803	43.838	33.259	34.990	31.308
5	80.144	80.399	69.724	70.641	80.260	70.629	71.163
6	95.550	492.19	232.81	501.14	391.93	60.128	380.34
7	374.65	537.66	500.79	535.32	567.03	362.80	530.46
8	512.32	692.10	483.34	682.19	516.70	513.88	520.13
9	526.30	525.38	495.20	715.92	706.98	529.09	533.60
10	8.9246	7.0096	16.318	19.882	21.053	13.583	8.4638
11	10.669	10.677	10.676	10.682	10.491	10.152	10.280
12	8.3375	8.5010	8.2992	8.6032	8.6268	8.0809	8.1215
13	561.83	628.58	592.13	627.86	632.37	450.91	684.90
14	11.161	12.106	11.584	12.168	16.934	8.8930	8.4244
15	27.235	186.46	71.325	182.01	22.273	74.564	34.921
16	35.629	115.82	75.511	214.90	198.62	12.002	74.948
17	800.38	598.86	570.28	597.53	602.04	582.26	600.15
18	5.0506	5.0722	5.0684	5.0725	5.0697	5.0461	5.0694
19	24.833	25.190	23.179	24.193	23.032	22.771	24.146
20	88.748	90.162	89.890	90.352	90.141	80.087	89.130

Table 7: PRE of the estimators for $n = 7$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	200.30	195.29	195.60	203.63	204.46	200.71	200.42
2	138.52	130.88	130.99	141.45	138.35	131.46	131.36
3	26.939	28.248	28.217	28.267	28.160	26.639	28.244
4	90.516	91.254	90.275	91.234	90.980	88.129	90.609
5	165.79	155.02	154.82	154.91	178.09	156.72	173.23
6	343.08	356.84	350.80	390.70	375.29	355.50	334.90
7	68.033	83.229	73.444	75.257	75.758	74.321	73.547
8	21.654	23.673	18.673	28.673	25.672	20.648	21.675
9	1.3041	1.3385	1.3380	1.3383	1.3357	1.2918	1.3429
10	4631.6	5789.9	1856.5	5791.9	4721.3	1203.5	1214.6
11	1437.1	1513.5	1511.8	2030.4	1893.5	677.02	702.77
12	587.96	975.96	609.28	903.13	851.48	833.31	609.66
13	14.166	15.413	15.469	15.442	15.782	14.195	15.430
14	142.50	189.62	228.22	248.93	156.41	149.48	143.77
15	54.751	29.682	44.761	32.909	66.351	57.910	51.851
16	19.279	24.506	23.504	25.993	23.684	12.369	12.213
17	4.0742	3.9214	3.9203	3.9249	3.9174	3.9080	3.9246
18	10.571	10.850	10.764	10.994	10.887	10.554	10.848
19	155.34	164.20	155.01	164.06	164.73	160.72	153.52
20	440.71	462.61	460.56	460.82	459.81	435.98	460.98

5.2 Results on the PRE

Results on the PRE of different estimators presented in tables 5, 6 and 7 show that their efficiency gain compared to the direct estimator $\bar{y}$ although noticeably high in most cases, in some cases it is just marginal. In some populations, the comparable estimators also perform very similarly in the sense that there is not any appreciable difference between their PRE values. On the consideration of the overall performance, the four estimators $\ell_P, \ell_{SP}, \ell_S$ and ℓ_{SS} are inferior to the other three estimators ℓ_{RP}, ℓ_{AP}

and ℓ_{NP}, and both ℓ_{SP} and ℓ_S remain as the worst performer whereas the performance of ℓ_P or ℓ_{SS} appears highly unsatisfactory.

From the results on the PRE values of the three competing estimators ℓ_{RP}, ℓ_{AP} and ℓ_{NP}, we see that ℓ_{AP} and ℓ_{RP} are decidedly the most efficient in 9 and 4 populations for all values of n whereas ℓ_{NP} is the same in 4 populations for $n = 3$, and in 5 populations for $n = 5$ and 7. On the other hand, ℓ_{AP} is the second most efficient in 6 populations for $n = 3$ and in 5 populations for $n = 5$ and 7, and ℓ_{NP} is the same in 5 populations for all values of n. In view of these findings and considering their performances, we directly rank ℓ_{AP}, ℓ_{NP} and ℓ_{RP} respectively as the best, second best and third best performers.

Table 8: Coverage rate of the estimators for $n = 3$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	81.95	80.87	80.77	80.90	80.87	81.92	80.85
2	74.03	73.84	73.72	73.79	73.81	74.65	73.65
3	59.72	59.72	59.72	59.76	59.72	59.72	59.72
4	76.68	75.86	75.78	75.70	75.91	76.65	75.80
5	81.76	81.89	81.67	81.85	81.96	81.50	81.89
6	11.80	11.73	11.30	11.73	11.73	11.19	11.73
7	18.52	18.52	18.52	18.52	18.52	18.52	18.52
8	23.44	23.41	23.41	23.41	23.41	23.44	23.41
9	24.26	24.20	24.20	24.20	24.20	24.26	24.20
10	13.77	15.00	13.77	18.94	14.12	14.64	15.17
11	35.78	35.43	35.43	30.43	37.71	44.64	44.73
12	30.35	30.43	30.43	30.26	31.31	35.08	34.82
13	17.72	17.41	17.29	17.39	17.41	17.72	17.41
14	78.42	78.85	78.85	79.12	79.47	80.35	80.17
15	16.07	18.75	18.75	18.75	18.75	17.71	18.78
16	15.70	28.42	28.33	28.33	28.33	13.59	28.33
17	18.56	18.58	18.44	18.54	18.54	18.52	18.54
18	52.30	51.65	51.79	51.86	51.76	52.10	51.31
19	83.96	84.64	84.58	84.58	84.52	83.73	84.64
20	90.00	90.40	90.40	90.34	90.34	90.06	90.28

5.3 Results on the Coverage Rate

The coverage rates of nominal 95% confidence intervals for $\overline{Y}$ dependent on different estimators are presented in tables 8, 9 and 10. Note that the results for 99% are not displayed since they confirm more or less the tendencies found in the case of 95%. On the ground of the achieved CRs we note that the CRs of all estimators are unpredictable and usually bear no resemblance to the nominal rates aimed at. In many cases, the CRs of the confidence intervals are very poor. This under coverage is probably because of the choice of a seriously biased common approximate variance estimator for the comparable estimators. Surprisingly, in most of the populations we also see that there is no indication of increase in the quality of all estimators when sample size is enlarged rather their quality deteriorates.

Table 9: Coverage rate of the estimators for $n = 5$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	90.37	89.29	89.28	89.29	89.25	89.82	88.96
2	86.40	86.51	86.49	86.70	86.42	86.60	85.99
3	49.20	49.30	49.30	49.30	49.30	49.20	49.30
4	76.47	74.55	74.29	74.57	74.40	75.86	74.01
5	88.76	88.74	88.71	88.74	88.96	88.64	88.40
6	23.07	23.07	23.07	23.07	23.07	23.09	23.07
7	21.42	21.42	21.42	21.42	21.42	21.42	21.42
8	38.88	38.88	38.88	38.88	38.88	38.88	38.88
9	38.88	38.88	38.88	38.88	38.88	38.88	38.88
10	21.50	30.00	30.00	30.00	11.97	12.00	23.24
11	14.93	15.92	15.92	15.91	14.68	15.55	14.74
12	17.15	17.74	17.65	17.80	17.08	17.21	17.06
13	18.75	18.75	18.75	18.75	18.75	18.75	18.75
14	79.47	73.16	72.52	73.31	72.17	71.41	73.77
15	37.45	37.50	37.50	37.50	37.50	37.50	37.50
16	52.10	52.10	52.10	52.10	52.10	52.14	52.10
17	18.72	18.75	18.70	18.71	18.72	18.74	18.68
18	48.12	47.78	47.83	47.81	47.83	48.21	46.65
19	90.29	90.24	90.23	90.24	90.20	89.91	90.28
20	91.59	90.74	90.72	90.79	90.62	90.45	90.72

Table 10: Coverage rate of the estimators for $n = 7$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	91.20	90.28	90.28	90.28	90.25	90.55	89.89
2	90.64	90.14	90.13	90.12	90.08	89.91	89.61
3	52.80	52.86	52.86	52.86	52.86	52.81	52.86
4	81.70	81.33	81.31	81.32	81.36	81.87	81.61
5	90.55	90.20	90.19	90.21	90.18	90.22	89.90
6	30.76	30.76	30.76	30.76	30.76	16.40	30.76
7	34.04	34.05	34.05	34.05	34.05	34.06	34.04
8	49.73	49.73	49.73	49.73	49.73	49.73	49.73
9	49.73	49.73	49.73	49.73	49.73	49.73	49.73
10	40.00	40.00	40.00	40.00	40.00	40.26	40.00
11	39.99	39.97	39.97	39.97	39.53	25.27	38.62
12	40.00	39.99	39.99	39.99	39.45	25.32	35.07
13	31.89	31.87	31.87	31.87	31.87	31.91	31.87
14	67.36	66.51	66.47	66.53	66.96	72.73	74.34
15	50.00	50.00	50.00	50.00	51.60	51.03	50.20
16	56.71	59.07	59.11	65.26	62.28	54.56	65.20
17	36.50	36.51	36.51	36.51	36.51	36.51	36.51
18	46.53	45.95	45.97	45.97	46.01	46.78	44.89
19	92.49	92.55	92.55	92.56	92.54	92.43	92.73
20	93.80	93.72	93.71	93.78	93.73	93.81	94.15

From the displayed results on the achieved CR one major conclusion available to us is that all estimators perform about equally well. For this reason, it is not straight forward to decide which estimator could be given preference over other under such a performance measure. Although, the classical product estimator ℓ_P appears superior to others in about four to five populations, its improvement is only marginal and not enough to take a concrete decision in favor of the estimator.

Table 11: SD of the Student t statistic of the estimators for $n = 3$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.9599	0.9918	0.9923	0.9926	0.9937	0.9769	1.0033
2	1.0334	1.0327	1.0331	1.0350	1.0324	1.0311	1.0300
3	2.1778	2.1859	2.1860	2.1837	2.1775	2.1855	2.1848
4	1.2510	1.2858	1.2926	1.2895	1.2795	1.2673	1.3083
5	0.9070	0.8831	0.8833	0.8837	0.8860	0.9198	0.9010
6	75.749	75.931	75.855	75.234	75.886	75.524	75.863
7	46.743	46.707	46.698	46.518	46.506	46.730	46.714
8	25.908	25.856	25.846	25.856	25.857	25.900	25.854
9	27.693	27.639	27.629	27.639	27.640	27.683	27.636
10	2.8373	3.1162	3.0404	3.1064	2.6623	2.7459	2.3312
11	2.5531	2.5488	2.5480	2.2219	2.4145	2.3146	2.2531
12	1.5928	1.5500	1.5484	1.4492	1.4388	1.8073	1.4503
13	32.161	32.089	32.088	32.081	32.088	32.141	32.178
14	0.9394	1.0958	1.1212	1.0841	1.1687	1.4799	1.5170
15	11.656	12.071	11.909	9.9084	10.020	13.983	7.7701
16	15.108	15.304	15.242	14.330	15.167	15.275	14.893
17	74.341	74.313	74.309	74.312	74.314	74.336	74.632
18	2.8167	2.8142	2.8085	2.8050	2.8070	2.7629	2.7618
19	0.8440	0.8537	0.8425	0.8422	0.8460	0.8425	0.8453
20	0.6904	0.7120	0.7129	0.7254	0.7165	0.7220	0.7119

Table 12: SD of the Student t statistic of the estimators for $n = 5$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.8137	0.8369	0.8372	0.8376	0.8387	0.8319	0.8499
2	0.9259	0.9282	0.9285	0.9317	0.9282	0.9268	0.9263
3	2.0245	2.0355	2.0356	2.0336	2.0244	2.0351	2.0341
4	1.1453	1.1620	1.1642	1.1609	1.1630	1.1506	1.1684
5	0.8158	0.8187	0.8033	0.8288	0.8332	0.8037	0.8202
6	74.407	74.602	74.530	73.954	74.561	74.155	74.521
7	45.446	45.431	45.424	45.321	45.429	45.431	45.438
8	25.593	25.548	25.549	25.548	25.541	25.584	25.546
9	27.843	27.797	27.791	27.797	27.798	27.833	27.793
10	2.9135	3.1830	3.1207	1.9749	2.7865	3.2532	2.3729
11	2.3210	2.3209	2.3204	2.0696	2.1764	1.7536	1.7108
12	1.5912	1.5445	1.5429	1.2905	1.3935	1.4733	1.0247
13	32.065	32.006	32.004	32.000	32.005	32.046	32.040
14	0.7484	0.8186	0.8268	0.8107	0.8594	1.1846	1.1080
15	11.573	11.958	11.820	9.8148	10.379	10.239	6.4186
16	15.470	15.499	15.449	14.324	15.374	14.399	14.953
17	70.142	70.118	70.115	70.117	70.118	70.134	70.113
18	2.5619	2.5722	2.5673	2.5004	2.5634	2.5665	2.5217
19	0.7863	0.7771	0.7770	0.7856	0.7780	0.7767	0.7780
20	0.6557	0.6693	0.6697	0.6802	0.6719	0.6686	0.6758

5.4 Results on the SD of the Student t Statistic (σ_t)

Numerical values of the SD of the Student$-t$ statistic calculated for different estimators *i.e.*, their σ_t −values are compiled in tables 11, 12 and 13. On the ground of this criterion, the general behavior of the estimators seems to be inconsistent and in most of the cases the calculated σ_t −values of the sampling distributions of the estimators have very large deviations from their respective theoretical values. Searching for an estimator better than others seems to be difficult owing to very erratic results in favor of the estimators except ℓ_{AP} and ℓ_{SS} which appear to be good competitors.

ℓ_{SS} turns out as the best performer in 6 populations for $n = 3$ and in 5 populations for $n = 5$ and 7 whereas ℓ_{AP} is the best performer in 6, 9 and 12 populations for $n = 3, 5$ and 7 respectively. ℓ_{AP} also remains as the second best performer in most of the cases. These findings clearly confirms that although the performance of ℓ_{AP} is not so significant for $n = 3$, its overall performance improves when n increases and calculated σ_t −value tends rapidly towards the theoretical value. On the average, the estimators ℓ_{AP} and ℓ_{SS} look to be better than others. However, ℓ_{AP} may be ranked in the first position whereas ℓ_{SS} in the second in respect of the consistency under sampling fluctuations. Taking into account of the performance, we also select ℓ_{NP} as the third consistent estimator.

Table 13: SD of the Student t statistic of the estimators for $n = 7$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.3202	0.3304	0.3302	0.3316	0.3302	0.3201	0.3306
2	0.4322	0.4417	0.4417	0.4438	0.4426	0.4251	0.4429
3	1.0049	1.0439	1.0435	1.0443	1.0444	0.9940	1.0448
4	0.8349	0.8425	0.8379	0.8447	0.8435	0.8284	0.8432
5	0.5929	0.5981	0.5981	0.6037	0.6006	0.5884	0.6014
6	4.0662	4.0673	4.0672	4.0648	4.0690	4.0831	4.0841
7	11.111	11.101	11.101	10.833	11.102	11.121	11.101
8	6.0158	6.0402	6.0397	6.0399	6.0389	6.0427	6.0106
9	5.2096	5.2099	5.2094	5.1899	5.1869	5.2088	5.2118
10	0.6635	0.6642	0.6643	0.7124	0.6644	0.6633	0.6639
11	0.6292	0.6293	0.6294	0.6302	0.6311	0.6160	0.6290
12	0.2729	0.2860	0.2842	0.2998	0.2860	0.2734	0.2862
13	10.236	10.236	10.236	10.234	10.235	10.236	10.230
14	0.4286	0.4244	0.4246	0.4332	0.4182	0.3710	0.3599
15	1.8954	1.8951	1.8951	1.7619	1.8877	1.8948	1.8148
16	3.3469	3.3610	3.3608	3.2510	3.3508	3.4221	3.2650
17	14.246	14.245	14.245	14.223	14.244	14.243	14.217
18	1.9325	1.9305	1.9231	1.8814	1.8210	1.9305	1.8875
19	0.5713	0.5840	0.5842	0.5910	0.5842	0.5836	0.5898
20	0.4310	0.4319	0.4319	0.4320	0.4321	0.4360	0.4362

5.5 Results on the Coefficient of Skewness ($\boldsymbol{\beta_1}$)

An examination of the computed values of the coefficient of skewness *i.e.*, β_1 coefficient given in tables 14, 15 and 16 reveals that the sampling distribution of the competing estimators very much deviate from normality. Of course, for $n = 7$ in most of the populations, the distributions of the estimators are not far away from the normality. Inconsistent behavior of the estimators for different considered values of n (except ℓ_P, ℓ_{AP} and ℓ_S) in the approach to symmetry cannot give any clear-cut idea on their overall relative performance.

For $n = 3$, although the distributions of ℓ_P, ℓ_{AP} and ℓ_S appear to be skewed, for increased sample size the skewness of the distributions of ℓ_P slowly approaches to zero whereas ℓ_{AP} and ℓ_S do it faster. After analyzing magnitudes of the deviations of the computed β_1 −values from zero for all populations and for all sample sizes, we straightforwardly rank the classical product estimator ℓ_P as the best estimator. On the same ground, we also rank ℓ_{AP} and ℓ_S respectively as the second best and third best estimators.

Table 14: Coefficient of skewness of the estimators for $n = 3$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.4064	0.4440	0.4324	0.4479	0.4387	0.3800	0.4512
2	0.2657	0.2815	0.2943	0.2744	0.2876	0.3056	0.3004
3	2.0996	2.0388	2.0377	2.0413	2.0407	2.0967	2.0407
4	2.2395	2.9453	3.0495	2.9197	2.9977	2.6952	3.1980
5	0.2222	0.3810	0.3984	0.3792	0.3847	0.2722	0.4058
6	2.4569	2.5497	2.4713	2.4530	2.5502	2.4810	3.0151
7	3.5455	3.4859	3.3747	3.1455	3.4844	3.5052	3.6202
8	1.4498	1.6571	1.6864	1.6533	1.6516	1.4623	1.6431
9	1.5384	1.7485	1.7784	1.7440	1.7430	1.5510	1.7347
10	1.4002	1.5577	1.4655	1.4403	1.4804	1.4115	2.2938
11	1.5955	1.7009	1.6982	1.6971	1.6745	1.5855	1.7741
12	1.3968	1.4196	1.4220	1.4211	1.4226	1.4433	1.4932
13	4.0108	4.0419	4.0537	4.0047	4.0344	3.9693	4.0240
14	1.0403	1.1724	1.1813	1.1811	1.2591	1.7055	1.7831
15	0.6557	0.6655	0.6571	0.5257	0.7762	0.8010	0.8165
16	1.5204	2.3613	1.7590	2.2849	2.0596	1.5067	6.2591
17	4.7942	4.9252	4.9751	4.7893	4.9203	4.8692	4.9252
18	3.4946	4.5138	4.1869	4.3798	4.5848	3.9815	5.5155
19	0.2871	0.3798	0.3819	0.3607	0.3883	0.3805	0.4083
20	0.3834	0.5331	0.5383	0.5340	0.5490	0.5212	0.5771

Table 15: Coefficient of skewness of the estimators for $n = 5$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.2974	0.3191	0.3147	0.3212	0.3158	0.2772	0.3231
2	0.1981	0.2063	0.2112	0.2019	0.2110	0.2362	0.2286
3	1.4118	1.3768	1.3763	1.3789	1.3779	1.4100	1.3780
4	1.6917	2.0876	2.1286	2.0694	2.1187	2.0193	2.2406
5	0.1873	0.2829	0.2899	0.2824	0.2860	0.2321	0.3091
6	1.7049	1.7020	1.6597	1.6461	1.6633	1.6716	1.9700
7	2.2365	2.3647	2.3141	2.1791	2.3647	2.2254	2.4513
8	1.1231	1.1256	1.1348	1.0132	1.1225	1.0209	1.1172
9	1.0760	1.1894	1.1991	1.0835	1.1863	1.1864	1.1812
10	0.8120	0.8939	0.8524	0.8313	0.8637	0.8233	1.4767
11	0.9365	0.9873	0.9864	0.9303	0.9761	0.9892	1.0438
12	0.8031	0.8134	0.8142	0.8144	0.8170	0.8489	0.8946
13	2.8397	2.8550	2.8592	2.8348	2.8511	2.8136	2.8441
14	0.6313	0.6860	0.6878	0.6923	0.7408	1.1594	1.2655
15	0.2675	0.2721	0.2684	0.2508	0.3697	0.4136	0.4185
16	0.9144	1.3631	1.0756	1.3165	1.2396	0.9040	3.5249
17	3.3256	3.3883	3.4059	3.3557	3.3856	3.3209	3.3827
18	2.9613	3.0328	2.8796	2.4596	3.0795	2.8704	3.7525
19	0.2173	0.2701	0.2709	0.2701	0.2748	0.2708	0.2882
20	0.2916	0.3738	0.3757	0.3751	0.3822	0.3738	0.3978

Table 16: Coefficient of skewness of the estimators for $n = 7$

Pop. No.	ℓ_P	ℓ_{RP}	ℓ_{SP}	ℓ_{AP}	ℓ_{NP}	ℓ_S	ℓ_{SS}
1	0.4282	0.5092	0.5072	0.4984	0.5138	0.4847	0.5631
2	1.1455	1.2786	1.2792	1.2544	1.2868	1.2375	1.3945
3	1.0709	1.0690	1.0691	1.0680	1.0690	1.0699	1.0692
4	0.3412	0.3645	0.3642	0.3669	0.3607	0.3235	0.3340
5	0.2081	0.3002	0.3022	0.2910	0.3061	0.2561	0.3292
6	0.0040	0.0047	0.0040	0.0028	0.0062	0.0497	0.0501
7	0.2079	0.2129	0.2121	0.2121	0.2236	0.2904	0.3003
8	1.3399	1.3429	1.3428	1.3409	1.3425	1.3384	1.3430
9	1.3396	1.3425	1.3424	1.3382	1.3422	1.3406	1.3426
10	0.0121	0.0153	0.0152	0.0116	0.0123	0.0122	0.0666
11	0.0001	0.0248	0.0248	0.0002	0.0176	0.0788	0.0247
12	0.0072	0.0078	0.0078	0.0256	0.0109	0.0060	0.0076
13	1.7195	1.7826	1.7775	1.7854	1.7814	1.7281	1.7887
14	0.0462	0.0553	0.0551	0.0455	0.0623	0.0831	0.1212
15	0.0784	0.0874	0.0872	1.9651	0.0001	0.0254	0.0257
16	0.5329	0.5509	0.5513	0.4145	0.5251	0.4113	0.5742
17	2.4195	2.5874	2.5900	2.5880	2.5751	2.3687	2.4875
18	1.7453	2.1886	2.0592	1.8941	2.1907	2.1583	2.4422
19	0.0052	0.0059	0.0059	0.0007	0.0053	0.0071	0.0097
20	0.0273	0.0282	0.0281	0.0255	0.0303	0.0482	0.0509

6. Conclusions

From the simulation study we see that the overall performance of the almost unbiased estimator ℓ_{AP} compared to others on the basis of the statistical measures *viz*., biasedness, efficiency and SD of the Student t Statistic is highly satisfactory. From the asymmetry point of view this estimator cannot compete with the classical product estimator ℓ_P and ranked as the second best estimator. But when n increases, ℓ_{AP} approaches more rapidly towards normality than ℓ_P. In view of this, we cannot rightly say that ℓ_{AP} is worse than ℓ_P since approach to symmetry is a large sample property of an estimator. Of course on the ground of the CR, performance of ℓ_{AP} is not so impressive but not at all discouraging as an estimator performs more or less similarly with its competitors. Hence, considering these findings we conclude that ℓ_{AP} is the best performer amongst all.

The results of this numerical study also leads to the following tentative conclusions:

(i) The proposed new almost unbiased estimator ℓ_{NP} may be preferred as the second best choice in respect of ARB and PRE, and third best choice in respect of SD of the Student t only.

(ii) ℓ_P is the most preferable only on the consideration of skewness

(iii) Sahoo & Sahoo's (1999) almost unbiased estimator ℓ_{SS} is the third and second best choices only in terms of ARB and SD of the Student t respectively.

(iv) Robson's (1957) unbiased estimator ℓ_{RP} and Srivastava's (1983) predictive product estimator ℓ_S are respectively the third best and second best choices on the basis of PRE and skewness.

In view of the above conclusions, even if the performance of an estimator on the CR is not taken into account, selection of an estimator as the second or third best performer is not feasible. This issue arises due to changeable behavior of an estimator for different populations with varying numerical characteristics in terms of the variables even if in respect of a specific performance indicator. Bad performance of the comparable estimators on their achieved CRs in most of the cases is of course discouraging. But, using Student's t –distribution in place of normal distribution can improve the CR to some extent.

Findings of this numerical evaluation are only expressive and the conclusions drawn may not necessarily be fitted to all populations. But they lay out certain guidance on the overall capabilities of the estimators under comparison. It is therefore required to continue further work with other populations to gather better idea on the relative performance of different estimators.

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